Indian J. Phys. **66B** (4), 407-415 (1992)

Relativistic filamentation and modulational instabilities of a laser radiation in a laser produced plasma

A A Mamun
Institute of Nuclear Science & Technology, Atomic Energy Research Establishment,
G P O Box 3787, Dhaka, Bangladesh
and
M Salimullah, F Majid and M N Alam
Department of Physics, Jahangirnagar University, Savar, Dhaka, Bangladesh

Received 16 January 1992, accepted 29 May 1992

Abstract : A theoretical investigation has been made on the relativistic filamentation and modulational instabilities of a laser radiation propagating in an unmagnetized collisionless laser-produced plasma. The relativistic Vlasov equation has been employed to find the nonlinear response of electrons for the four-wave parametric processes. It is noted here that for typical plasma parameters of interest the relativistic growth rates of both the filamentation and modulational instabilities turn out to be quite large and increase with increasing the intensity of the incident laser radiation and the equilibrium electron density.

Keywords : Filamentation and modulational instabilities, relativistic Vlasov equation, growth rate, laser produced plasma, dispersion relation.

PACS Nos. : 52.40. Nk, 52.60. +h, 52.35. Mw

1. Introduction

A beam of high power laser radiation propagating in a plasma created by the light itself irradiating a target parametrically interacts with the low frequency electrostatic perturbation mode present in the plasma. This parametric interaction process involves two types of nonlinearities. The first one comes from the radiation pressure effects i.e., ponderomotive force on electrons, whereas the second arises from the relativistic effects i.e., relativistic electron mass variation. These nonlinearities cause the parametric amplification of the low frequency electrostatic perturbation mode which may result the filamentation or modulational instability of the incident laser beam depending up on the direction of propagation of the perturbation mode. If the growing perturbation propagates in a direction transverse to the direction propagation of the incident laser beam, the instability results in the breaking up of the incident laser beam into filamentary structures and is known as filamentation instability. Again, if the growing perturbation propagates along the direction of the incident laser beam and the phase velocity of the perturbation mode is equal to the

Our new format of references has been adopted in this paper.

group velocity of the incident beam, the instability results in modulating the amplitude of the incident laser beam and is called the modulational instability. The filamentation and modulational instabilities have been studied in gaseous and solid state plasmas with or without magnetic fields in recent years [1-8]. The available literature on these phenomena deals with the nonrelativistic description of plasma which is valid only up to the moderate power density of the laser radiation. With the technological advancement more and more high-power lasers are being designed. In the presence of high power laser beams, the electrons in the plasma acquire high quiver velocities comparable to the free-space velocity of light. In such cases, the response of electrons cannot be described by nonrelativistic plasma equations and the effects of relativistic electron mass must be taken into account. A limited number of attempts have been made using the fluid equations to investigate the relativistic effects in the laser plasma interactions [9, 10]. In Ref. [9] Shukla *et al* have studied shortly the relativistic nonlinear effects on the filamentation and modulational instabilities by using fluid model. In their report the growth rate is found to be directly proportional to the intensity of the incident beam. To the best of our knowledge there is no rigorous theoretical study on relativistic filamentation and modulational instabilities of a laser radiation in a laser produced plasma by kinetic theory. In this paper, by employing kinetic theory we have made a rigorous theoretical investigation of the filamentation and modulational instabilities of a laser radiation by a low frequency purely electrostatic perturbation mode which may be present in the laser produced plasma due to the presence of electron plasma wave, ion acoustic mode or some other reasons. Since we consider the case of relativistic and hot plasmas, we have employed kinetic approach but not fluid model.

In Section 2 we have solved the full relativistic Vlasov equation to obtain the nonlinear density fluctuation associated with the low frequency electrostatic perturbation mode due to the nonlinear mode coupling of the incident laser radiation with the generated electromagnetic sidebands in a hot homogeneous unmagnetized laser produced plasma. In Section 3 we have derived the nonlinear dispersion relation for the low frequency perturbation mode and obtained the growth rates for both the filamentation and modulational instabilities. The numerical evaluation of the results is presented in the same section (Section 3). Finally, a brief discussion of the results is given in Section 4.

2. Nonlinear response of electrons

We consider the propagation of a large amplitude linearly polarized laser radiation of angular frequency ω_0 and propagation vector $\underline{k}_0 \parallel \hat{z}$ in a hot homogeneous unmagnetized collisionless laser produced plasma. The oscillatory electric and magnetic fields $\underline{E}_0$ and $\underline{B}_0$ of the incident laser radiation (pump wave) can be expressed as

$$\underline{E}_0 = \underline{E}'_0 \exp[-(\omega_0 t - k_0 z)],$$
$$\underline{B}_0 = c\underline{k}_0 \times \underline{E}_0/\omega_0\,, \tag{1}$$

where c is the speed of light in vacuum and the propagation constant k_0 is equal to the vacuum wavelength [11, 12] :

$$k_o \approx \omega_o/c. \tag{2}$$

Now, we consider the existence of a low frequency purely electrostatic density perturbation (ω, $\underline{k}$) which may be present in the hot plasma due to any low frequency mode, e.g., electron plasma wave. Due to the interaction of the pump wave (ω_o, $\underline{k}_o$) with this perturbation mode (ω, $\underline{k}$), two high frequency electromagnetic side-bands ($\omega_{1,2} = \omega \mp \omega_o$, $\underline{k}_{1,2} = \underline{k} \mp \underline{k}_o$) are generated. These sidebands in turn interact with the pump wave and produce a low frequency ponderomotive force which then amplifies and drives the electrostatic perturbation mode. Thus, we consider the four wave parametric decay of a laser beam (ω_o, $\underline{k}_o$) into the two scattered electromagnetic waves ($\omega_{1,2}$, $\underline{k}_{1,2}$) and the electrostatic perturbation wave (ω, $\underline{k}$) associated with the electron plasma wave.

In the presence of the high frequency laser radiation and the decay waves, the response of electrons to the four-wave parametric decay processes in the hot plasma may be suitably described by the relativistic Vlasov equation [13].

$$\frac{\partial f^T}{\partial t} + \frac{1}{\gamma}\, \underline{v}.\, \underline{\nabla} f^T - \frac{e}{m}\left(E^T + \frac{1}{\gamma c}\, \underline{v} \times \underline{B}^T\right).\, \underline{\nabla}_v f^T = 0, \tag{3}$$

where

$$\underline{v} = \gamma(\dot{x}\,\hat{x} + \dot{y}\,\hat{y} + \dot{z}\,\hat{z}),$$

$$\gamma = (1 + v^2/c^2)^{1/2},$$

the superscript T represents the total quantity involved and the overdot refers to the time derivative of the quantity.

The total distribution function can be expressed as

$$f^T = f_o^{\,o} + f_o(\omega_o, \underline{k}_o) + f(\omega, \underline{k}) + f_1(\omega_1, \underline{k}_1) + f_2(\omega_2, \underline{k}_2), \tag{4}$$

where f is the distribution function corresponding to the perturbation mode (ω, $\underline{k}$), $f_i(\omega_i$, $\underline{k}_i)$; $i = 0, 1, 2$ are the distribution functions corresponding to the pump wave (ω_o, $\underline{k}_o$), lower sideband (ω_1, $\underline{k}_1$), and upper sideband (ω_2, $\underline{k}_2$), respectively, and $f_o^{\,o}$ is the equilibrium distribution function taken to be Maxwellian form at temperature T_e :

$$f_o^{\,o} = n_o^{\,o}\,(m/2\pi k_B T_e)^{3/2} \exp(-mv^2/2k_B T_e), \tag{5}$$

where $n_o^{\,o}$ is the equilibrium electron density, m is the rest mass of the electron, v is the random speed of the electrons and k_B is the Boltzmann constant.

We now express

$$\underline{E}^T = \underline{E}_o + \underline{E} + \underline{E}_1 + \underline{E}_2,$$

$$\underline{B}^T = \underline{B}_o + \underline{B}_1 + \underline{B}_2, \tag{6}$$

and make use of the approximations, $\gamma\omega_o > \underline{k}_o.\underline{v}$, $\gamma\omega_{1,2} > \underline{k}_{1,2}.\,\underline{v}$ to solve eq. (3) and obtain the linear response functions of electrons corresponding to the pump wave (ω_o, $\underline{k}_o$), generated sidebands ($\omega_{1,2}$, $\underline{k}_{1,2}$) and the perturbation mode (ω, $\underline{k}$), respectly, as

$$\frac{ief_o^{\,o}}{k_B T_e \omega_o} \cdot (\underline{E}_o \cdot \underline{v})\left(1 + \frac{\underline{k}_o \cdot \underline{v}}{\gamma\omega_o}\right),$$

$$f_1 \quad \frac{\cdot\, {}^{o}}{k_B T_e \omega_1} \cdot (\underline{E}_1 \cdot \underline{v}) (1 + \frac{\underline{k}_1 \cdot \underline{v}}{\gamma\omega_1}),$$

$$f_2^L \simeq -\frac{ief_o^{\,o}}{k_B T_e \omega_2} \cdot (\underline{E}_2 \cdot \underline{v}) (1 + \frac{\underline{k}_2 \cdot \underline{v}}{\gamma\omega_2}), \tag{7}$$

$$f^L \quad \frac{ief_o^{\,o}}{k_B T_e \omega} \cdot (\underline{E} \cdot \underline{v}) (1 + \frac{\underline{k} \cdot \underline{v}}{\gamma\omega})^{-1}.$$

The nonlinear response of electrons for the perturbation mode $(\omega, \underline{k})$ can be obtained by solving the nonlinear relativistic Vlasov equation of the form :

$$\frac{\partial f^{NL}}{\partial t} + \frac{1}{\gamma}\underline{v} \cdot \underline{\nabla} f^{NL} = \frac{e}{m}(\underline{E}_1 \cdot \underline{\nabla}_v f_o^{\,L} + \underline{E}_2 \cdot \underline{\nabla}_v f_o^{\,L*} + \underline{E}_o \cdot \underline{\nabla}_v f_1^{\,L*} + \underline{E}_o^{\,*} \cdot \underline{\nabla}_v f_2^{\,L}$$
$$+ \frac{1}{\gamma c}\underline{v} \times \underline{B}_1 \cdot \underline{\nabla}_v f_o^{\,L} + \frac{1}{\gamma c}\underline{v} \times \underline{B}_2 \cdot \underline{\nabla}_v f_o^{\,L*}$$
$$+ \frac{1}{\gamma c}\underline{v} \times \underline{B}_o \cdot \underline{\nabla}_v f_1^{\,o} + \frac{1}{\gamma c}\underline{v} \times \underline{B}_o^{\,*} \cdot \underline{\nabla}_v f_2^{\,L}). \tag{8}$$

Here, the asterisk* denotes the complex conjugate of the quantity involved. Since the incident laser radiation is polarized in the x-direction, the electrons will be almost constrained to quiver in this direction. Therefore, to keep the mathematics manageable without loosing any insight of the problem, we may take $\gamma = (1 + v^2/c^2)^{1/2} \simeq v_x/c$.

Now, under the approximations, $\gamma\omega_o > \underline{k}_o \cdot \underline{v}$, $\gamma\omega_{1,2} > \underline{k}_{1,2} \cdot \underline{v}$, $\gamma\omega < \underline{k} \cdot \underline{v}$ and $\gamma >> 1$, one can solve eq. (8) and obtain the nonlinear distribution function associated with the perturbation mode $(\omega, \underline{k})$, f^{NL}. The integration of this nonlinear distribution function f^{NL} over the velocity space gives the nonlinear density fluctuation associated with the perturbation mode $(\omega, \underline{k})$, n^{NL}. This nonlinear density fluctuation n^{NL} contains the x and z components of the electric fields associated with the generated waves $(\omega_{1,2}, \underline{k}_{1,2})$, $E_{1x,2x}$ and $E_{1z,2z}$. To evaluate $E_{1x,2x}, E_{1z,2z}$ we have to substitute the nonlinear current densities associated with the generated waves $(\omega_{1,2}, \underline{k}_{1,2})$:

$$\underline{J}^{NL}_{1,2} = -e \int \frac{\underline{v}}{\gamma} f^{NL}_{1,2}\, d\underline{v}, \tag{9}$$

where $f_{1,2}^{\,NL}$ are the nonlinear distribution functions corresponding to generated sidebands $(\omega_{1,2}, \underline{k}_{1,2})$ and are respectively, given by

$$f_1^{NL} = \frac{ie}{2m\omega_1}(1 + \frac{\underline{k}_1 \cdot \underline{v}}{\gamma\omega_1}) \left[\underline{E} \cdot \underline{\nabla}_v f_o^{\,L*} + (\underline{E}_o^{\,*} + \frac{1}{\gamma c}\,\underline{v} \times \underline{B}_o^{\,*}).\underline{\nabla}_v f^L\right],$$

$$f_2 \quad = \frac{\quad}{2m\omega_2} \quad \frac{k_2 \cdot v}{\gamma\omega_2} \left[\underline{E} \cdot \underline{\nabla}_v f_o^{\,L} + (\underline{E}_o + \frac{1}{\gamma c}\,\underline{v} \times \underline{B}_o).\underline{\nabla}_v f^L\right], \tag{10}$$

into the wave equations for the generated waves $(\omega_{1,2}, \underline{k}_{1,2})$:

$$D_{1,2} \cdot \underline{E}_{1,2} = \frac{4\pi i \omega_{1,2}}{c^2} \underline{J}^{NL}_{1,2}. \tag{11}$$

Here $D_{1,2} = (k_{1,2}^2 - \frac{\omega_{1,2}^2}{c^2} E_{1,2}) I - \underline{k}_{1,2}\underline{k}_{1,2}$ are the dispersion tensors, $E_{1,2} = 1 - \frac{\omega_p^2}{\omega_{1,2}}$ are the linear dielectric functions of the high frequency sidebands $(\omega_{1,2}, \underline{k}_{1,2})$, $\underline{I}$ is the unit tensor of rank two, and $\omega_p = (4\pi n_o^o e^2/m)^{1/2}$ is the electron plasma frequency. The substitution of these evaluated fields $E_{1x,2x}$ and $E_{1z,2z}$ into the expression for n^{NL} gives the final equation for n^{NL}.

Therefore, the nonlinear density fluctuation n^{NL} associated with the perturbation mode $(\omega, \underline{k})$ for the filamentation instability, i.e., for the case $\underline{k} \perp \underline{k}_o$ is finally given by

$$n^{NL} = \frac{e^3 |E_o|^2 n_o^o \omega_p^2 k_1^4 \Phi}{2m^3 c^2 v_e^4 \omega_o^2} \left(\frac{P_1}{|D_1|} + \frac{P_2}{|D_2|} \right). \tag{12}$$

Here

$$P_1 = \frac{k_{1z}^2}{k_1^2} R_1 \left[\frac{ck_{1z}}{\omega_1} A_1^o \left(\frac{ck_{1z}}{\omega_1} R_1^x + \frac{k_{1z}}{k_{1x}} L_1 \right) - \left(\frac{ck_{1x}}{\omega_1} A_1^o - \frac{\omega_o}{\omega_1} A_1^z \right) \left(\frac{ck_{1z}}{\omega_1} \cdot \frac{k_{1z}}{k_{1x}} + \frac{k_{1z}^2}{k_{1x}^2} R_1^z L_1 \right) \right],$$

$$P_2 = \frac{k_2^2}{k_1^2} \cdot \frac{k_{2x}^2}{k_1^2} R_2 \left[\frac{ck_{2z}}{\omega_2} A_2^o \left(\frac{ck_{2z}}{\omega_2} R_2^x + \frac{k_{2z}}{k_{2x}} L_2 \right) - \left(\frac{ck_{2x}}{\omega_2} A_2^o - \frac{\omega_o}{\omega_2} A_2^z \right) \left(\frac{ck_{2z}}{\omega_2} \cdot \frac{k_{2z}}{k_{2x}} + \frac{k_{2z}^2}{k_{2x}^2} R_2^z L_2 \right) \right],$$

$$\begin{aligned}
L_{1,2} &= 1 + \frac{ck_{1x,2x}}{\omega_{1,2}} + \frac{ck_{1z,2z}}{\omega_{1,2}} \left(1 \pm \frac{k_o}{k_{1,2}}\right), \\
R_{1,2} &= 1 - \frac{\omega_{1,2}^2}{c^2 k_{1,2}^2} \varepsilon_{1,2}, \\
R^x_{1,2} &= 1 - \frac{\omega_{1,2}^2}{c^2 k_{1x,2x}^2} \varepsilon_{1x,2x}, \\
R^z_{1,2} &= 1 - \frac{\omega_{1,2}^2}{c^2 k_{1z,2z}^2} \varepsilon_{1z,2z}, \\
A^o_{1,2} &= 1 \pm \frac{\omega_o}{\omega_{1,2}}, \\
A^z_{1,2} &= 1 \mp \frac{ck_{1z,2z}}{\omega_{1,2}},
\end{aligned} \tag{13}$$

where the upper sign corresponds to the first quantity and the lower sign corresponds to the second quantity involved. In eq. (12), Φ is the electrostatic potential of the perturbation mode $(\omega, \underline{k})$ which is assumed to be purely electrostatic i.e., $\underline{E} = - \underline{\nabla}\, \Phi$, $v_e = (2k_B T_e/m)^{1/2}$ is the thermal speed of electrons, $|D_{1,2}|$ are the determinants of the dispersion tensors $D_{1,2}$.

Similarly, the nonlinear density fluctuation n_m^{NL} associated with the perturbation mode $(\omega, \underline{k})$ for the modulational instability, i.e., for the case $\underline{k} \| \underline{k}_o$ is finally given by

$$n_m^{NL} = \frac{e^3 |E_o|^2 n_o^o \omega_p^2 k_1^4 \Phi}{m^3 c^2 v_e^4 \omega_o^2} \left(\frac{P'_1}{|D_1|} + \frac{P'_2}{|D_2|} \right), \tag{14}$$

where

$$P'_1 = A_1 R_1 \left[\frac{\omega_1}{ck_1} \varepsilon_1 A_1^{\circ} - R_1 \left(A_1^{\circ} + \frac{ck_1}{\omega_1} S_1\right) \right],$$

$$P'_2 = A_2 R_2 \left[\frac{\omega_2}{ck_2} \varepsilon_2 A_2^{\circ} - R_2 \left(A_2^{\circ} + \frac{ck_2}{\omega_2} S_2\right) \right] \left(\frac{k_2^4}{k_1^4}\right), \tag{15}$$

$$A_{1,2} = 1 + \frac{ck_{1,2}}{\omega_{1,2}},$$

$$S_{1,2} = 1 \mp 2\frac{\omega_o}{\omega_{1,2}},$$

and the other quantities $A^{\circ}_{1,2}$, and $R_{1,2}$ are given in eqs. (13).

3. Growth rates of the instabilities

In order to obtain the growth rate of the instability, we have to derive the nonlinear dispersion relation for the perturbation mode $(\omega, \underline{k})$. Substituting the nonlinear electron density fluctuation n^{NL} associated with the perturbation mode $(\omega, \underline{k})$ in to Poisson's equation

$$\varepsilon \Phi = -\frac{4\pi e}{k^2} n^{NL}, \tag{16}$$

where ε is the dielectric function of the electrostatic perturbation mode $(\omega, \underline{k})$ which under the condition $\omega < kv_e$, is given by [4, 14]

$$\varepsilon \simeq 1 + \frac{2\omega_p^2}{k^2 v_e^2}\left(1 + \frac{2\omega^2}{k^2 v_e^2}\right), \tag{17}$$

we obtain the nonlinear dispersion relation for the perturbation mode $(\omega, \underline{k})$ in the case of filamentation instability as

$$\frac{\mu_1}{|D_1|} + \frac{\mu_2}{|D_2|}. \tag{18}$$

Here $\mu_{1,2}$ are the coupling coefficients given by

$$\mu_{1,2} \simeq -\frac{|V_o/c|^2 \omega_p^4 k_1^4}{2k^2 v_e^4} P_{1,2}, \tag{19}$$

and $|V_o| = e\,|E_o|/m\omega_o$ and $P_{1,2}$ are given in eqs. (13).

When the parametric resonance conditions [4], $\omega_{1,2} = \omega \mp \omega_o$, $\underline{k}_{1,2} = \underline{k} \mp \underline{k}_o$ are satisfied, we can expand ε and $|\underline{D}_{1,2}|$ as

$$\omega = \omega_r + i\gamma_g,$$

$$\varepsilon = i(\gamma_g + \gamma_L)\left(\frac{\partial \varepsilon}{\partial \omega}\right), \tag{20}$$

$$|\underline{D}_{1,2}| = i(\gamma_g + \gamma_{L1,L2})\left(\frac{\partial |\underline{D}_{1,2}|}{\partial \omega_{1,2}}\right),$$

where γ_g is the overall growth rate of the instability and γ_L, γ_{L1}, and γ_{L2} are the damping rates of the perturbation mode $(\omega, \underline{k})$, and the high frequency sidebands $(\omega_{1,2}, \underline{k}_{1,2})$, respectively. In our study, we neglect the linear damping of the decay waves in the collisionless limit of the laser-produced plasma. However, one could write the expressions for the threshold of the instability by considering the effect of collisions in the corresponding dispersion relation of the decay waves. Thus, using eqs. (20), the growth rate of the instability in the absence of linear damping of the decay modes can be writen in the form

$$\gamma_o^2 = -\frac{1}{\frac{\partial\varepsilon}{\partial\omega}}\left[\frac{\mu_1}{\frac{\partial|D_1|}{\partial\omega_1}} \cdot \frac{\mu_2}{\frac{\partial|D_2|}{\partial\omega_2}}\right] \tag{21}$$

Therefore, on simplification the growth rate γ_o of the relativistic filamentation instability, where $k \perp k_o$, is given by

$$\gamma_o = \frac{1}{4\sqrt{2}}|V_o/c|\,\omega_p\left(\frac{ck}{\omega}\right)\left(\frac{\omega P_1}{\varepsilon_1\omega_1 R_1 R'_1} \cdot \frac{\omega P_2}{\varepsilon_2\omega_2 R_2 R'_2} \cdot \frac{k_1^4}{k_2^4}\right)^{1/2} \tag{22}$$

where

$$R'_{1,2} = 1 - 3\frac{\omega_{1,2}}{c^2 k_{1,2}^2}, \tag{23}$$

$P_{1,2}$ and $R_{1,2}$ are given in eqs. (13).

Similarly, the growth rate γ_o of the relativistic modulational instability is given by

$$\gamma_o = \frac{1}{4}|V_o/c|\,\omega_p\left(\frac{\omega P'_1}{\varepsilon_1\omega_1 R_1 R'_1} + \frac{\omega P'_2}{\varepsilon_2\omega_2 R_2 R'_2} \cdot \frac{k_1^4}{k_2^4}\right)^{1/2}, \tag{24}$$

where $R_{1,2}$, $P'_{1,2}$ and $R'_{1,2}$ are given in eqs. (13), eqs. (15) and eq. (23), respectively.

The final expressions for the growth rates of filamentation and modulational instabilities in relativistic consideration, eqs. (22) and (24) have the physical meaning that the growth rates in both the instabilities depend on $|V_o/c|$, i.e., the intensity of the incident laser radiation, ω_p i.e., on the equilibrium electron density (n_o^o), and on the fundamental parameters $(\omega, \omega_{o,1,2}, \underline{k}, \underline{k}_{o,1,2}, \varepsilon_{1,2})$ of the interacting and generated beams. It may be mentioned here that the growth rates in both the instabilities increase with the intensity of the incident beam and also with the equilibrium electron density when all other parameters remain constant and the approximation $\omega_{1,2} \geq \omega_p$ is valid.

In order to have some numerical evaluation of the resuls of the present theory, we hace made calculations of the relativistic growth rates of the filamentation and modulational instabilities for the following typical plasma parameters : $n_o^o = 10^{15}$— 10^{16}cm^{-3}, $|V_o/c| =$ 0.1 — 1.0, $T_e = 1$ KeV, $\omega_o = 1.778 \times 10^{14}$ rad. sec^{-1}(corresponding to CO_2 laser), and $k =$ 1.18×10^4 cm^{-1}.

The results of these numerical calculations are shown in the form of curves drawn in the adjoining figure (Figure 1).

In the figure, the upper two curves (A and A') show the variation of the growth rates (normalized to the pump frequency) γ_0/ω_0 of both the filamentation and modulational instabilities in relativistic situation as a function of the normalized and directed pump induced drift velocity of electrons, $|V_0/c|$ and the lower two curves (B and B') show the variation of the same γ_0/ω_0 as a function of the equilibrium density of electrons, n_0° for different plasma parameters of interest. The solid curves (A and B) are for filamentation instability and the dotted curves (A' and B') are for modulational instability. The left and upper scale is for the upper curves (A and A) and the left and lower scale is for the lower curves (B and B'). The relativistic growth rate in both the instabilities increases with increasing $|V_0/c|$ and also with n_0° as long as the approximation $\omega_p \leq \omega_{1,2}$ is valid.

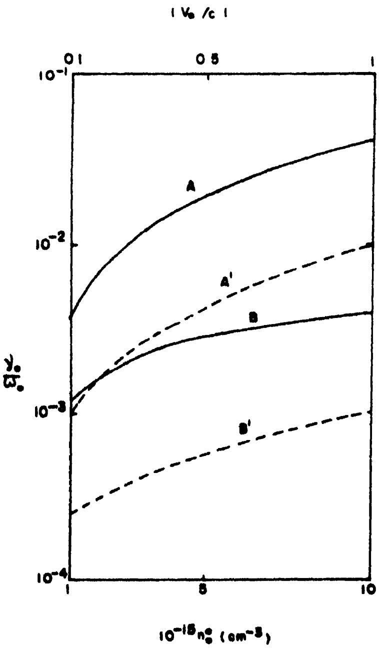

Figure 1. The variation of γ_0/ω_0 as a function of $|V_0/c|$ with constant $n_0^{\circ} = 10^{16}$ cm^{-3} and as a function of n_0° with constant $|V_0/c| = 0.5$ for $T_e = 1$ KeV, $\omega_0 = 1.778 \times 10^{14}$ rad. sec^{-1} and $k = 1.18 \times 10^4$ cm^{-1}. The upper two curves (A and A') represent the variation of γ_0/ω_0 with $|V_0/c|$ and the lower two curves (B and B')represent the variation of γ_0/ω_0 with n_0°. The solid curves represent the filamentation instability while the dashed curves represent the modulational instability. The left and lower scale are for curves B and B' while the left and upper scale are for the curves A and A'.

4. Discussion

A high power linearly polarized laser radiation propagating in an unmagnetized collisionless laser produced plasma is efficiently unstable against the relativistic filamentation and modulational instabilities. Here, it is observed that the nonlinearities for the generation of the instabilities have been considered through the ponderomotive force on electrons and the relativistic electron mass variation. The ponderomotive force on electrons and the relativistic effects on electron mass are responsible for the parametric amplification of the low frequency electrostatic perturbation mode which may cause the filamentation or modulational instability of the incident laser beam depending upon the direction of propagation of the perturbation mode. For typical plasma parameters : $n_0^\circ = 10^{15}$ cm^{-3}, $|V_0/c| = 0.1$, $T_e = 1$ KeV, $\omega_0 = 1.778 \times 10^{14}$ rad. sec^{-1} (corresponding to CO_2 laser), $k = 1.18 \times 10^4$ cm^{-1} the relativistic growth rate is $\sim 10^{11}$ rad. sec^{-1} for filamentation instability and is $\sim 3 \times 10^{10}$ rad. sec^{-1} for modulational instability. It can be noticed here that the relativistic growth rate γ_0 for both the filamentation and modulational instabilities increases with increasing the normalized and directed pump induced drift velocity of electrons ($|V_0/c|$) i.e., the intensity of the incident beam which is in agreement with Refs. [4, 9] and also increases with the increasing electron plasma frequency i.e., the equilibrium electron density (n_0°) which is also in agreement with Refs. [4, 5]. The results of this paper differ from those of Refs. [4, 5, 9] that the numerical values of the growth rates in both the instabilities are very large in comparison with those of Refs. [4, 5, 9]. It may be noted that the kinetic theory gives more accurate results than the fluid model does and both the filamentation and modulational instabilities have large relativistic effects.

It may be noted here that the saturation of all possible parametric instabilities at the extreme relativistic situation, the effects of the self generated megagauss magnetic fields, and the inhomogeneities in the self-generated magnetic field and the plasma produced by the laser itself are also problems of great importance, but beyond the scope of the present paper.

References

[1] J Drake, P K Kaw, Y C Lee, G Schmidt, C S Liu and M N Rosenbluth 1974 Phys. Fluids 17 778

[2] R L Stenzel 1976 *Phys. Fluids* 19 865

[3] A Salzmann, D Ng and A A Offenberger 1979 *Phys. Rev. Lett.* 43 1502

[4] C S Liu and P K Kaw 1976 *Advances in Plasma Physics* Vol 6 ed W B Thompson, (New York : Wiley p 4, 83, 91

[5] Md Salimullah and Tersem Singh 1982 *J. Phys. Chem. Solids* 43 1087

[6] R S Tiwari and S R Sharma 1980 *Phys. Lett.* A77 30

[7] M Lisak 1980 *J. Plasma Phys.* 24 445

[8] Md Salimullah and F Majid 1982 *Phys. Rev.* A25 555

[9] P K Shukla, N N Rao, M Y Yu and N L Tsintzsadze 1986 *Phys. Rep.* 138 1

[10] P K Shukla, R Bharathram and M N Tsintzsadze 1988 *Phys. Scripta* 38 578

[11] P Kaw and J M Dawson 1970 *Phys. Fluids* 13 472

[12] C E Max 1971 *Phys. Fluids* 16 1277

[13] D E Baldwin, I B Bernstein and M P H Weekink 1976 *Advances in Plasma Physics* Vol. 3 eds A Simon and W B Thompson (New York : Wiley p. 4

[14] S Ichimaru 1986 *Plasma Physics* (California : The Benjamin/Cummings) p 278, 279